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Math Camp - Final Exam — Matthew Draper

1.

	Yes	No	$a \cdot a$	$\sqrt{a \cdot a}$
Liberal -	98	71	6958	83.41
Labour -	9	15		
Conserv. -	1	206		
Others -	0	3	206	14.35
(Abstain) -	267			

Vector Norm

$\|a\| = \sqrt{a \cdot a}$ $\Rightarrow$ Conservatives > Liberals

$\sqrt{1^2+206^2} > \sqrt{98^2+71^2}$

−1.5

2. (1 Day)

Mine # 1 20c, 550s

Mine # 2 30c 500

a) $5v_1$ = Five days' (operation) production from Mine #1 ✓

−0.5

b) $x_1 V_1 + x_2 V_2 = B$ $\left(B = \begin{bmatrix}150\\2825\end{bmatrix}\right)$ $\begin{bmatrix}20 & 30\\550 & 500\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}150\\2825\end{bmatrix}$

(3) $\begin{bmatrix}20\\550\end{bmatrix}$ (3) $\begin{bmatrix}30\\500\end{bmatrix}$ $\begin{bmatrix}150\\2825\end{bmatrix}$ $x_1 = x_2$

$\begin{bmatrix}90\\1500\end{bmatrix}$ 6.1 Days

c) Order 2×1 (except for A (2×2)) −0.5

3. $|A| = \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\1 & 8 & 9\end{bmatrix}\begin{matrix}1 & 2\\4 & 5\\1 & 8\end{matrix} = (\overset{45}{1\times5\times9}) + (\overset{60}{2\times5\times6}) + (\overset{96}{3\times4\times8}) - (\overset{15}{1\times5\times3}) - (\overset{48}{8\times6\times1}) - (\overset{72}{9\times4\times2})$

$|A| = 18$ ✓

4. $x_1\begin{bmatrix}0\\1\\5\end{bmatrix} + x_2\begin{bmatrix}1\\2\\8\end{bmatrix} + x_3\begin{bmatrix}4\\-1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$?

↳ 0 × anything = 0

Linearly Independent ✓

5. $AC + BC = (A+B)C$

$$AC = \begin{bmatrix} 3 & -5 \\ 13 & 11 \\ 8 & 3 \end{bmatrix} \qquad (A+B) = \begin{bmatrix} 3 & 0 \\ 2 & 4 \\ 5 & 8 \end{bmatrix}$$

$$BC = \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix} \qquad (A+B)C = \begin{bmatrix} 3 & 0 \\ 2 & 4 \\ 5 & 8 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

$3 \times 2 \quad 2 \times 2$

$$AC + BC = \begin{bmatrix} 3 & 9 \\ 10 & 2 \\ 21 & 7 \end{bmatrix} \qquad (A+B)C = \begin{bmatrix} 3 & 9 \\ 10 & 2 \\ 21 & 7 \end{bmatrix}$$ ✓

6. $$|B| = \begin{bmatrix} -r & a & 0 \\ 0 & -r & a \\ 1 & 0 & -r \end{bmatrix} \begin{matrix} -r & a \\ 0 & -r \\ 1 & 0 \end{matrix}$$

$-r^3 + 2a + 0 - (-r) - (-ra) - (-ra) =$

$|B| > 0$ $\quad = -r^3 + 2a + 2ra + r = |B| =$

$\Leftrightarrow$ FOR THE DETERMINANT $|B|$ TO BE POSITIVE,

$r^2 - r^3 > 0$. $(-r^3)$ MUST BE NEGATIVE, WHICH MEANS

-0.5. r MUST BE POSITIVE. r MUST ALSO BE $> a$

7. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x) = x - 2x^2 + 3x^4$

$f'(x) = \lim_{h \to 0} \frac{(x+h) - 2(x+h)^2 + 3(x+h)^4}{h}$

$= \lim_{h \to 0} \frac{(x+h) - 2(x^2 + 2xh + h^2) + 3(x+h)^4}{h}$

$= \lim_{h \to 0} \frac{(x+h) - 2x^2 - 4xh - 2h^2 + 3(x+h)(x+h)(x+h)(x+h)}{\cancel{h}}$

$= \lim_{h \to 0} 12x^3 - 4x^2 + 1$ ✓

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8a) $f(x) = x^2 - x^3 + 2$

$f'(x) = 2x - 3x^2$; $f(1) = -1$ ✓

b) $f(x) = \sqrt{x}\,(2x^2 - 5)$

$$f'(x) = \frac{d(f(x)\,g(x))}{dx} = \frac{d f(x)}{dx} g(x) + f(x)\frac{d g(x)}{dx}$$

$$= \frac{d(\sqrt{x})}{dx}(2x^2-5) + \sqrt{x}\,\frac{d(2x^2-5)}{dx}$$

$$= \frac{1}{2\sqrt{x}}(2x^2-5) + \sqrt{x}\,(4x) = \frac{10x^2-5}{2\sqrt{x}}$$

$$f(4) = \frac{10(4)^2 - 5}{2\sqrt{(4)}} = \frac{10(16)-5}{2(2)} = \frac{155}{4}$$ ✓

DIFFERENT f

c) $f(x) = \dfrac{4x^4 - 2x}{3x^2}$ $\quad f'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

$$f'(x) = \frac{(12x^3-2)(3x^2) - (4x^4-2x)(6x)}{3x^4}$$

$$= \frac{12x^3-2}{3x^2} - \frac{(4x^{\cancel{4}}-2x)(6x)}{3\cancel{x^4}} = \frac{12x^3-2}{3x^2} - \frac{24x-12x}{3}$$

$$= \frac{12x^3-2}{3x^2} - \frac{4x}{1} = \frac{8x^3+2}{3x^2}$$

$$f(-1) = \frac{8(-1)^3+2}{3(-1)^2} = \frac{-6}{3} = -2$$ ✓

9. $f(x) = x^3 - 9x^2 - 21x$

$f'(x) = 3x^2 - 18x - 21$

$f''(x) = 6x - 18$

FIRST DERIVATIVE TEST

$f'(x) = 0 = 3x^2 - 18x - 21$

$3x^2 - 18x = 21$ $\qquad$ $3x^2 - 18x - 21 = 0$

$x(3x - 18) = 21$ $\qquad$ $x(3x - 18 - \frac{21}{x}) = 0$

$\frac{1}{3}x\,(x - 6) = 7$

$3x^2 - 18x = 21$ $\qquad$ $FOC = f'(x = 7) = 0$

$x^2 - 6x = 7$ $\qquad$ $(7, -245)$

$x(x - 6) = 7$ $\qquad$ $= f'(x = 0) = -21$

$x = 7$

$(-1, 11)$

SECOND DERIVATIVE TEST

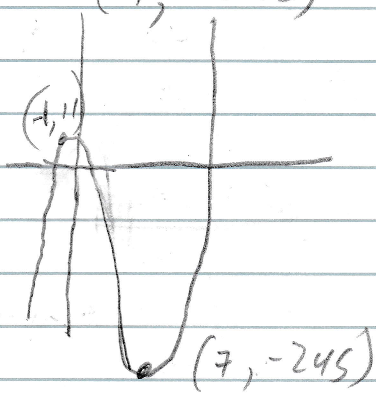
$f''(x) = 6x - 18$ $\qquad$ $(7, -245)$

$f''(7) = 6(7) - 18 = 24$

✓ $\Rightarrow (7, -245)$ IS A MINIMUM

$f''(-1) = -6 - 18 = -24$

$\Rightarrow (-1, 11)$ IS A MAXIMUM



10. a) $\int (3x^2 + x - 4)\,dx$

✓ $= x^3 + \frac{1}{2}x^2 - 4x + C$

b) $\int \frac{2x}{x^2 + 25}$ $\quad$ $du = x^2 + 25$

$v = \frac{1}{3}x^3 + 25x + C$ $\qquad$ -0.5

$= 2\ln(|x|) + 25x$ $\qquad$ $\ln(x^2 + 25) + C$

c) $\int x \ln x \, dx$: $f(x) = \frac{1}{\log(x)}$ $\quad$ $f'(x) = \frac{x\ln(a)}{1}$

-1 $\quad$ $= x\ln(x)$ (DOESN'T CHANGE)

$\frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C$

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11a) $\int_1^3 (2x+5)$; $\int(2x+5) = x^2+5x+C$

$(3) = 9-15 = -6$ $\quad (3)-(1) = (-6)-(-4)$

$(1) = 1-5 = -4$ $\quad = -2$

18?

-6.5.

b) $\int_{-2}^{-1}\left(\frac{2}{x^2}\right)$; $\int\left(\frac{2}{x^2}\right) = \frac{2}{1} \times \frac{1}{x^2} = \frac{-2}{x} + C$

$(-1) = 2$ $\quad 2-1 = 1$

$(-2) = 1$

✓ $\frac{du}{dx} = 2e^{2x}$

$u = e^{2x}+1$

c) $\int_0^1 \frac{5e^{2x}}{(1+e^{2x})^{\frac{1}{3}}}$ $\quad \int_a^b f(g(v)) \cdot g'(v)\, dv = \int_{g(a)}^{g(b)} f(t)\, dt$

$v' = (1+e^{2x})^{\frac{1}{3}} = \sqrt[3]{1+e^{2x}}$ $\quad \int 5e^{2x} = \frac{5e^{2x}}{2}$

$v = \frac{e^{2x}}{6} + x + C$

-1.5.

$\int \frac{5e^{2x}}{(1+e^{2x})^{\frac{1}{3}}} = \frac{\frac{5e^{2x}}{2}}{\frac{e^{2x}}{6}} = \frac{\frac{5e^{2x}}{2}}{\frac{1}{3}\left(\frac{e^{2x\frac{1}{3}}}{2}\right)} = 5\int \frac{e^{2x}}{e^{2x\frac{1}{3}}}$

$(1) = e^2 / (e^2)^{\frac{1}{3}}$

$(0) = e^0 / (e^0)^{\frac{1}{3}} = \frac{1}{1} = 1$ $\quad (1)-(0) = e^{2^{\frac{1}{3}}} - 1$

12. $f(x,y) = 3x^2y - 6xy^4 + (x+y)(x^3-y^2)$

$= 3x^2y - 6xy^4 + x^4 + xy^2 + x^3y + y^3$

$\partial/\partial x = 6xy - 6y^4 + 4x^3 + y^2 + 3x^2y$ ✓

$\partial/\partial y = 3x^2 - 24xy^3 + 2xy + x^3 + 2y^2$ ✓

$\frac{\partial^2 x}{\partial x^2} = 6y - 12x^2 + 6xy$ ✓

$\frac{\partial^2 y}{\partial y^2} = -72xy^2 + 2x + 4y$ ✓

13. $f(x,y) = (x-1)^2 + y + 1$

$= x^2 - 2x + y + 2$

✓ $\nabla f = \begin{bmatrix} 2x-2 \\ 1 \end{bmatrix}$ $\quad 2x - 2 = 0$

$x = 1 \quad Y = -1$

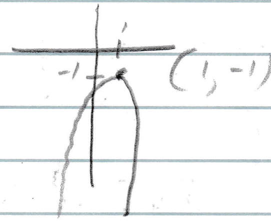
$(1, -1)$

✓ $H = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ MAXIMUM

$\begin{bmatrix} 2-\lambda & 0 \\ 0 & -\lambda \end{bmatrix}$ MINOR : 2 (+)

$2 \times 0 = 0$

$0 \times 0 = 0$



$\Rightarrow$

$\lambda = 0, 2 \geq 0$. CONVEX $\quad -1$

$\Rightarrow$ saddle point.

14a)

b) $A(s) = 100$ ✓

$B(s) = 40$

$A'(s) = -200 \sin(x+s)$ $\quad -0.5$.

$B'(s) = 30(m-3)^2 = 30 \cdot 2^2 = 120$.

c) ? $\quad -1$

15. ~~PE~~ MAXIMUM, NOT MINIMUM

$p' = \dfrac{8.4(t^2+49) - 2t(8.4t)}{(t^2+49)^2}$. $\quad -3$

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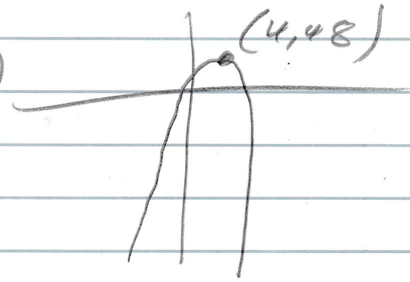
MATTHEW DOZIER

16. $V(T) = 30 + 12T^2 + (-T)^3 \quad 0 < T < 8$

$V'(T) = 24T - 3T^2$

MAXIMUM $= (4, 48)$



-11.

39/50

PROBABILITY

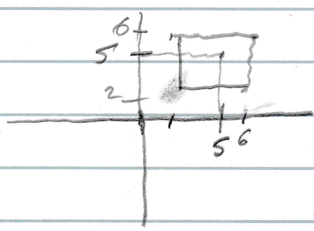
3/4

1a) $\Omega = \begin{array}{ccc} S, & G, & C \end{array} \begin{bmatrix} H \\ T \end{bmatrix}\begin{bmatrix} H \\ T \end{bmatrix}\begin{bmatrix} H \\ T \end{bmatrix} = \{ S_H G_H C_H,\ S_H G_H C_T,\ S_H G_T C_T,$
$2 \times 2 \times 2 = 8$ $\quad S_H G_T C_H,\ S_T G_H C_H,\ S_T G_H C_T,$
$S_T G_T C_H,\ S_T G_H C_T \}$

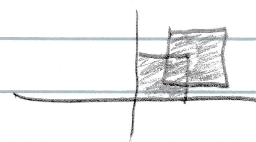
b) A (TWO HEADS) $= \{ S_H G_H C_T,\ S_H G_T C_H,\ S_T G_H C_H \}$

2. $A(D_1 = \text{EVEN} \wedge D_2 = \text{EVEN})$ a) NONE ARE MUTUALLY EXCLUSIVE
$B(D_1 + D_2 \geq 9)$ (TWO SIXES SATISFY ALL 3 CONDITIONS)
$C(D_1 \geq 5 \cap D_2 \geq 5)$ b) $C \in B$ (PERFECT SUBSET)

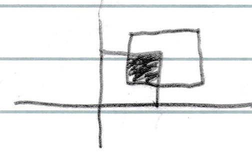
3.



a) $A \cup B$



b) $A \cap B$



3/4

4. A (CHIP IS EVEN) ; $P(A) = \frac{18}{36}$
$B(B/3 = R(0))$; $P(B) = \frac{12}{36}$

a) $P(A \cup B) = P(A) + P(B) = \frac{30}{36} - \cancel{\frac{6}{36}} = \frac{24}{36}$ This.

b) $P(A \cap B) = 6, 12, 18, 24, 30, 36 = \frac{6}{36}$

1/4 x5. $P(\text{HEART}) = 13/52$ $\quad 13 \times 4 = 52$ WAYS

$P(\text{KING}) = 4/52$

3/4 6. $P(\text{L BEATS E}) = .3$

$P(\text{L BEATS M}) = .2$

$P(\text{L BEATS NO-ONE}) = .6$

$P(\text{L BEATS BOTH}) = .1$

P(L Beats Both) is in both of these already

$P(\text{L BEATS E or M}) = P(\text{L BEATS E}) + P(\text{L BEATS M})$

$- P(\text{L BEATS BOTH}) = .3 + .2 - .1 = .4$ $= .3$

Need to subtract this twice

2/4 7. $P(A \cap B^c) = .2$ $\quad P(A \cup B \mid (A \cap B)^c) = ?$

$P(A^c \cap B) = .3$ from these we know

$P((A \cup B)^c) = .1$ $\quad P((A \cap B)^c) = .6$

BAYES: $\dfrac{P(A \mid B)P(B)}{P(A \mid B)P(B) + P(A \mid \sim B)(P(B))}$

$P((A \cap B^c) \cup (A^c \cap B)) \;/\; P((A \cap B)^c)$

$(.2) \cup (.3)$ $\quad .6$

$.5 / .6 = 5/6$

2/4 8a) $\binom{10}{1}\binom{10}{1} = 10 \times 10 = 100$ ✓

b) $\binom{10}{4}\binom{10}{4} = \left(\dfrac{10!}{4!\,6!}\right)^2 = 210 \times 210 = 44{,}100$ ✓

× 4! ways to arrange them into pairs

c) $\binom{20}{2} = \dfrac{20!}{2!\,18!} = 190$

$\binom{20}{4}$ two pairs of skaters = 4 total

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9. I II

6r 10w 3r 3w

3/4 a) GIVEN R, $P(II) = P(RED - I) * P(RED - II) = \frac{3}{16}$ P(RED)

$P(RED) = 9/25$ $6/16$ $3/6$

A = TOOK FROM URN II

B = DREW A RED CHIP

You solved the reverse of the question. We want P(A|B) as you defined them, not P(B|A)

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)} = \frac{\left(\frac{1}{3}\right)\left(\frac{9}{25}\right)}{\left(\frac{1}{2}\right)}$$

$$= \frac{6}{25}$$

1/4 x b) $P(B|A) = \frac{\left(\frac{5}{14}\right)\left(\frac{14}{30}\right)}{1/3} = \frac{\frac{1}{6}}{1/3} = \frac{1}{2}$ III 5r, 3w

3/4 10. $b(900, 1/200, 2) = \binom{900}{2}\left(\frac{1}{200}\right)\left(\frac{199}{200}\right)^{898} = .1122$

Complement of these three

$b(900, 1/200, 1) = \binom{900}{1}\left(\frac{1}{200}\right)\left(\frac{199}{200}\right)^{899} = .0497$

$b(900, 1/200, 0) = \binom{900}{0}\left(\frac{1}{200}\right)\left(\frac{199}{200}\right)^{900} = .0109$

$b(900, 1/200, 0) = \binom{900}{3}\left(\frac{1}{200}\right)\left(\frac{199}{200}\right)^{897} = 0.1688$

0.3416

11. a) $PMF = \sum P(y_i) = 1$

b)

c) $\int (x-\mu)^2 f(x)\,dx$

SORRY, RAN OUT OF TIME It happens!

12a) $E(x) = \sum_i x_i \left(P(x = x_i)\right)$

$E(x) = e^{-\nu} \nu e^{\nu}$

b) $V(x) = \int_{-\infty}^{\infty} (x-\nu)^2 f(x)\, dx.$

$\sigma = \sqrt{V(x)}$

+2 13.

	A	B
S	0	-5 (100%)
W	0	5 (100%)

✓ a) IF S(.25) + W(.75), THEN

$E(W) = .25(-5) + .75(5) = 2.5$

b) $P(B|A) = \dfrac{P(A|B)P(B)}{P(A)}$

14. $F(x) = x^{1/2}$

CDF: $P(Y \le y) = \sum_{i \le y} P(i)$

$P(X \le x) = F(x) = \int_0^x f(t)\, dt$