9/17/18

Math Camp Day 5 - Homework Matthew Draper

✓ 1(a) $\binom{15}{6} = \frac{15!}{6!(15-6)!} = \frac{15!}{720 \times 9!} = \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10}{720} = 5005$ ✓

b) $\binom{10}{3} \times \binom{5}{3} = \left(\binom{10}{3} = \frac{10!}{3!(7)!} = 120\right)\left(\binom{5}{3} = \frac{5!}{3!(2!)} = 10\right) = 1200$ ✓

c) $\binom{10}{4} \times \binom{5}{2} = \left(\binom{10}{4} = \frac{10!}{4!(6!)} = 210\right)\left(\binom{5}{2} = \frac{5!}{2!(3!)} = 10\right) = 2100$ ✓

✓ 2. Urn: 5w, 4b, 3r; Draw 4 w/o replacement

$P(W_1, R_2, W_3, B_4) = \frac{5}{12} \times \frac{3}{11} \times \frac{4}{10} \times \frac{4}{9} = \frac{240}{11880} = \frac{2}{99}$

✓ 3. 2-Stage Process (Bayes); Draw 1 from I, place in II

I	II	
2W, 1R	1W, 2R +	2W, 2R 2W, 2R 1W, 3R

If red is drawn from II, what is P(transfer & W)?

$P(W|R) = \frac{P(R|W)P(W)}{P(R|W)P(W) + P(R|\sim W)P(\sim W)} = \frac{(\frac{1}{2})(\frac{2}{3})}{(\frac{1}{2})(\frac{2}{3}) + (\frac{3}{4})(\frac{1}{3})} = \frac{(\frac{1}{3})}{(\frac{1}{3})(\frac{1}{4})} = \frac{4}{7}$

✓ 4. Each encounter with a customs agent can be modeled as a Bernoulli trial where the agent can choose to demand a bribe (failure) or perform honestly (success). $P(\text{success}) = .95$ $\Omega = \{S, F\}$

$b(n,p,k) = \binom{n}{k} p^k q^{(n-k)}$ $p = .95,\ q = .05$

$b(20, .95, 17) = \binom{20}{17}(.95^{17})(.05^3) = 1140(.418)(.000125) = .059582$

$b(20, .95, 18) = \binom{20}{18}(.95^{18})(.05^2) = 190(.397214)(.0025) = .188677$

$b(20, .95, 19) = \binom{20}{19}(.95^{19})(.05^1) = 20(.377354)(.05) = .377354$

$b(20, .95, 20) = \binom{20}{20}(.95^{20})(.05^0) = 1(.358486)(1) = .358486$

$P(T \geq 17) = P(T=20) + P(T=19) + P(T=18) + P(T=17) = .984099$

We can say that we would observe at least 17 successes in 20 trials 98.4% of the time.

✓ 5. $\binom{25}{12} = 5{,}200{,}300$ $\quad \dfrac{\binom{2}{1}\binom{23}{11} + \binom{2}{2}\binom{23}{10}}{\binom{25}{12}} =$

$= \dfrac{(2)(1{,}352{,}078) + (1{,}144{,}066)}{5{,}200{,}300}$

$P(\text{HUNG JURY}) = 37/50 = .74$

✓ 6. $b(12{,}000, .000125 \;(1/8000), 0) = \binom{12000}{0}\left(\frac{1}{8000}\right)^0\left(\frac{7999}{8000}\right)^{12000} = 0.2231$

$b(12{,}000, \frac{1}{8000}, 1) = \binom{12000}{1}\left(\frac{1}{8000}\right)^1\left(\frac{7999}{8000}\right)^{11{,}999} = 0.3347$

$b(12{,}000, \frac{1}{8000}, 2) = \binom{12000}{2}\left(\frac{1}{8000}\right)^2\left(\frac{7999}{8000}\right)^{11{,}998} = 0.2510$

$b(12000, \frac{1}{8000}, 3) = \binom{12000}{3}\left(\frac{1}{8000}\right)^3\left(\frac{7999}{8000}\right)^{11{,}997} = 0.1255$

$P(\text{HOSPITAL RUNS OUT}) = 1 - P(\text{HOSPITAL DOES NOT RUN OUT})$

$P(\text{HOSPITAL DOES NOT RUN OUT}) = \sum_{i=3}^{0} b(K=0\text{-}3) = .9353$

$P(\text{HOSPITAL RUNS OUT}) = 1 - .9353 = .0657$

$P(X=k) \approx \frac{\lambda^k}{k!}e^{-\lambda}$ $\quad P(x=0) \approx 1.5^0 \times e^{-1.5} = 0.2231$

$\lambda =$ $\quad P(x=1) \approx 1.5^1 \times e^{-1.5} = 0.3347$

$K=3$ $\quad P(x=2) \approx 1.5^2 \times e = 0.2510$

$P(x=3) \approx 1.5^3 \times e = 0.1255$

0.0657

3/4 7. PAYOFF IF $(H) = (2^1 + 2^2 + 2^3 \ldots 2^n)$

EXPECTED RETURN (KTH FLIP) $= \lim_{n\to\infty} = \frac{2(2^1+2^2\ldots 2^n)}{2}$

$\frac{1}{2} \times \$2 = 1$ $\quad \sum_{k=1} 2^k p(1-p)^{k-1}$

$\frac{1}{4} \times \$4 = 1$ $\quad \lim_{k\to\infty} S_1 = 2 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^n$

$\frac{1}{8} \times \$8 = 1$ $\quad$ = NO LIMIT. DOESN'T CONVERGE

$\frac{1}{16} \times \$16 = 1$ $\quad$ (AN EXPECTED UTILITY FUNCTION

$\frac{1}{32} \times \$32 = 1$ $\quad$ COULD SOLVE THIS, BUT WE WOULD NEED TO KNOW THE VALUE OF A MARGINAL \$)

Expected payoff of playing the game is ∞, so you will pay any amount less than this (or equal, if such a transaction is possible).

YOU ARE INDIFFERENT BETWEEN OFFERING THE GAMBLER ANY AMOUNT OF DOLLARS AND NOTHING AT ALL, B/C THE EXPECTED RETURN IS EXACTLY INDEXED TO THE PROBABILITY OF SUCCESS (ST. PETERSBURG PARADOX).

[redacted]

9/21/18

Math Camp Day 6 – Homework

Matthew Draper

1a) $\mu = E(x) = \int_a^b x f(x)\,dx$ or $\sum_{x \in S_x} x_i f(x)$

$P(k) = \dfrac{\binom{20}{k}\binom{60}{10-k}}{\binom{80}{10}}$

$\sum x\, m(x) = -1(0.935) + 2(0.0514) + 18(0.0115) + 180(0.0016) + 1300(1.35\times10^{-4}) + 2600(6.12\times10^{-6})$
$+ 10{,}000(1.12\times10^{-7}) = -.9356 + .1028 + .2070$
$+ .2880 + .1755 + .0519 + .0011 = -\cancel{.1087}\ .14468$

$-.1087 \times \$1 = E(x) \approx \$-.14$

b) $V(x) = \sum (x-\mu)^2 m(x)$ $\quad \alpha = -.1087 \quad 1-\mu = 1.1087$

$V(x) = E((x-\mu)^2) = (-1-(-.1087))^2 \times (0.935) + (2-(-.1087))^2 \times (.0514) + (18+.1087)^2 \times (.0115) + (180+.1087)^2 \times (.0016) + (1300+.1087)^2 \times (1.35\times10^{-4}) + (2600+.1087)^2 \times (6.12\times10^{-6}) + (10{,}000+.1087)^2 \times (1.12\times10^{-7}) = .7428 + .2286 + 3.7711 +$
$51.9026 + 228.1880 + 41.3747 + 11.2002 =$
$= \sigma^2 = V(x) = 337.408$

1/4 See my email tonight.

2. $\sigma = \sqrt{V(x)}$; $V(x) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$; $\mu = 0$

$0 \le y \le 1$ $\quad V(x) = \int_0^1 = V(1) - V(0) = 0(.5) = 0$

$2 \le y \le 3$ $\quad V(x) = \int_2^3 \quad V(3) - V(2) = 2(.5) = 1$

0 otherwise $\quad V(x) = \int_{-\infty}^{\infty}$ – DOES NOT CONVERGE

$\sigma = \sqrt{0} = 0$ $\qquad V(y) = \int_0^1 (y - \frac{3}{2})^2 \frac{1}{2}\,dy + \int_2^3 (y - \frac{3}{2})^2 \times \frac{1}{2}\,dy$

$\sigma = \sqrt{1} = 1$

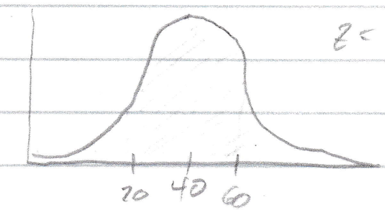
I don't follow all of this.

3. $E(X-a)^2 = E\left(\left((X-\mu)+(\mu-a)\right)^2\right)$
$= E\left((X-\mu)^2\right) + E\left((\mu-a)\right)^2 +$
$2(\mu-a)E(X-\mu)$

$= \text{Var}(X) + (\mu-a) \; ; \; E(X-\mu)=0$
$\text{Min}\,(a=\mu)$ of $g(a) = \text{Var}(X)$

✓

4. $\phi(x) = \frac{x-\mu}{\sigma} = \frac{x-40}{\sigma}$
$P(20 \le Y \le 60) = .5$ when
Z-score $(60) \times 2 = \frac{60-40}{\sigma}$

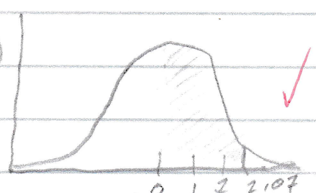


$z < .6745$

$\int_{20}^{60} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$

$\sigma = 29.65$

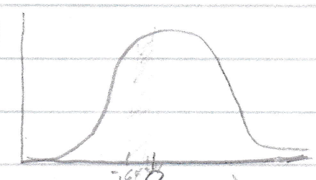
✓ 5a)



✓ $\mu = 0$
$\sigma^2 = 1$
(standard normal)

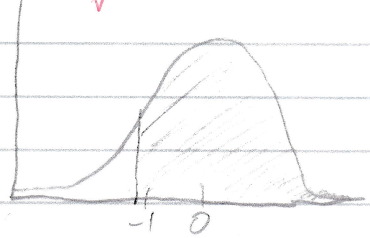
$P(0 \le z \le 2.07) = .4808$

✓ b)



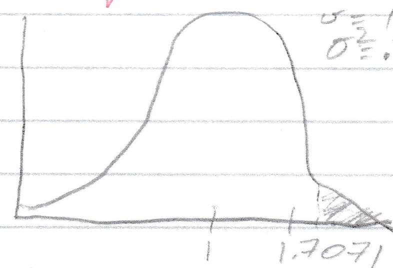
$P(-.64 \le z \le -.11) = P(-.64$
$\le z \le 0) - P(-.11 \le z \le 0)$
$= .2389 - .0438 = .1951$

c) ✓



$P(z \ge -1.06) = .8554$

✓ d)



$\mu = 1$
$\sigma^2 = .721$

$P(z > 2) = .0787$